Online Appendix to "Time-Varying Variance Decomposition of Macro-Finance Term Structure Models"

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Main paper is available here

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B Empirical Appendix

This appendix shows the general-to-specific model selection results. I also report results from misspecification testing showing that the model is well specified. Parameter estimates are reported in Tables 2–3.

B.1 Model specification

First, I test for the lag length of the macroeconomic variables under the physical measure given $K = 2$ terms in the conditional variance matrix. Table 1 shows information criteria and likelihood-ratio tests for general-to-specific model selection. The maximum number of lags considered is $L = 15$. The upper panel of the table unambiguously shows that an extensive lag structure is needed to match the data. The model with $L = 13$ lags obtains the minimum values of Akaike and Hannan-Quinn information criteria and close to the minimum value of the Bayesian information criterion. There are large and significant improvements in the in-sample fit with $L = 13$ compared to the choice of an annual lag length of $L = 12$.

With three latent factors and two macroeconomic variables, the model is identified with up to $K = 3$ components in the conditional variance matrix, see part (ii) of Proposition 1 of the main paper. In the lower panel of Table 1, I compare models with $K = \{1, 2, 3\}$ for the selected lag length of $L = 13$. I observe a large increase in the log-likelihood value when choosing $K = 2$ in comparison with the simplest model with $K = 1$. This difference persists when punishing for the additional parameters as reflected by all information criteria. Moreover, the difference is statistically significant. Thus, as explained in the main paper, it is necessary to include multiple components in the conditional variance matrix to fit the data. In contrast, the improvement in fit obtained by selecting $K = 3$ is modest and not statistically significant. As a result, $K = 2$ is sufficient to match the data.

B.2 Misspecification testing

Figure 1 shows the standardized residuals, their histograms superimposed with the Gaussian density, and autocorrelation functions. The figure also shows autocorrelation functions of the absolute values of standardized residuals, which serve as a test of the ability

Lag Length L	Variance Terms K	Log-Likelihood	AIC	BIC	HQC	LR-Test P-Value
15	2	1384.37	−2526.73	**−2763.04**	−2658.77	1.00
14	2	1382.40	−2530.80	−2759.10	−2658.47	1.00
13	2	1380.93	**−2535.86**	−2756.17	**−2659.17**	0.00
12	2	1345.23	−2472.46	−2684.77	−2591.41	0.01
11	2	1340.03	−2470.05	−2674.36	−2584.63	0.27
10	2	1339.41	−2476.82	−2673.13	−2587.04	0.95
9	2	1341.07	−2488.14	−2676.44	−2593.99	0.96
8	2	1342.77	−2499.54	−2679.84	−2601.02	0.00
7	2	1335.89	−2493.77	−2666.08	−2590.89	0.32
6	2	1335.53	−2501.07	−2665.37	−2593.82	0.00
5	2	1328.01	−2494.03	−2650.33	−2582.42	0.49
4	2	1328.29	−2502.59	−2650.89	−2586.61	0.00
3	2	1315.63	−2485.26	−2625.57	−2564.92	0.91
2	2	1317.13	−2496.26	−2628.57	−2571.56	0.00
1	2	1306.98	−2483.95	−2608.26	−2554.88	–
13	1	1239.82	−2273.64	−2473.95	−2386.04	0.00
13	2	1380.93	**−2535.86**	−2756.17	−2659.17	–
13	3	1387.38	−2528.75	**−2769.06**	**−2662.98**	0.23

Table 1: The table shows criteria for model selection of the physical dynamics with respect to lag length L and number of terms in the conditional variance matrix K. The upper panel varies L for fixed $K = 2$ while the lower panel varies K for fixed $L = 13$. Information criteria are the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQC). The minimum values of these criteria in each panel are boldfaced. The likelihood ratio tests in the upper panel are conducted against the model with $(L - 1)$. In the lower panel, the models are compared to the model with $K = 2$ using likelihood ratio tests.

of the conditional variance matrix to eliminate ARCH effects. The standardized residuals appear to be Gaussian and the autocorrelation functions of both levels and absolute values are mostly insignificant. There is, however, some autocorrelation left in the residual of the equation for inflation.

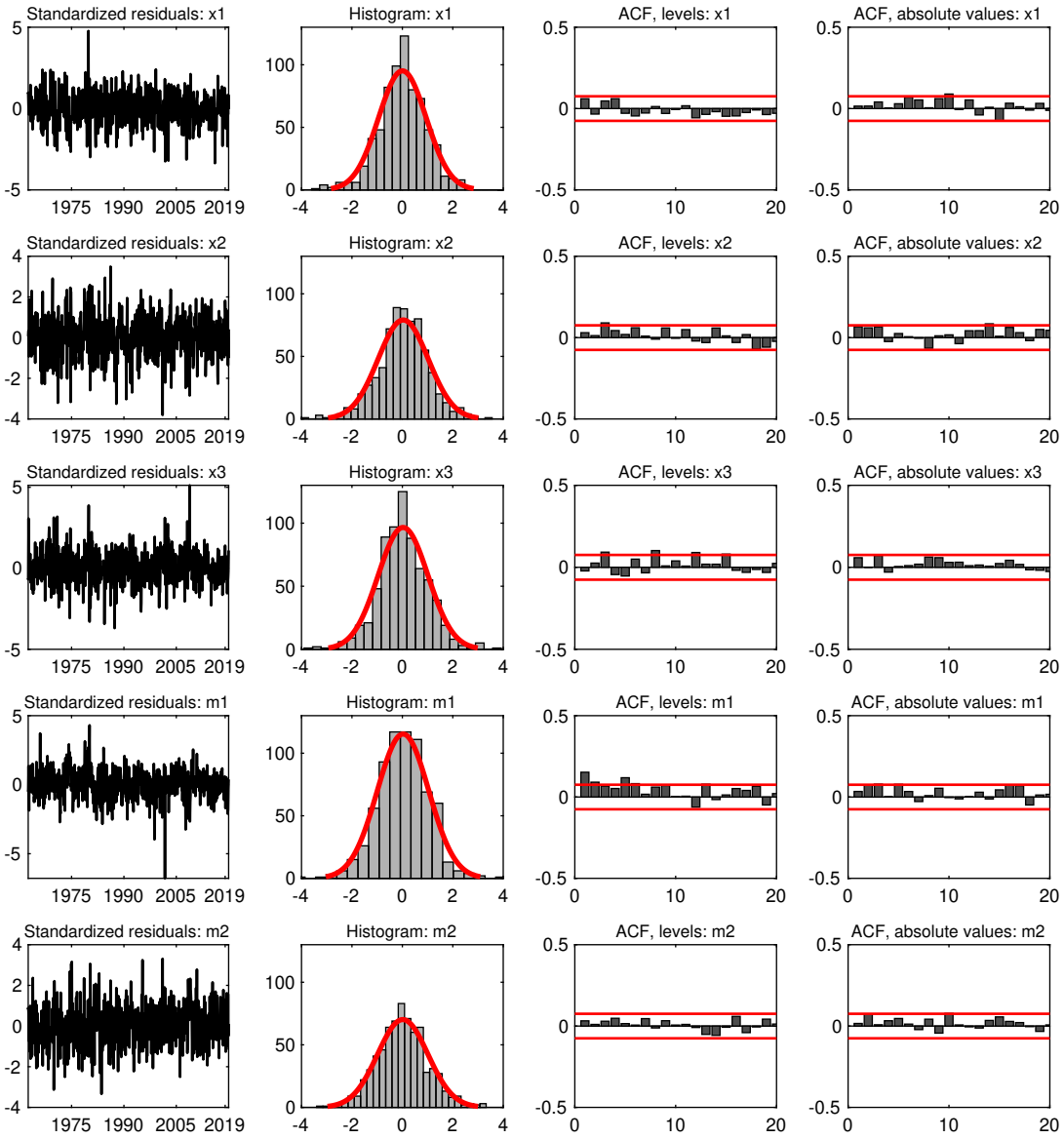

Figure 1: The figure shows misspecification testing results of the dynamics under the physical probability measure. The first column shows the standardized residuals. Charts in the second column exhibit histograms of the standardized residuals with a Gaussian density function superimposed. The third and fourth columns show autocorrelation functions of, respectively, levels and absolute values of standardized residuals with 95% confidence intervals computed under the assumption that the standardized residuals are Gaussian white noise. The variables x1, x2, and x3 refer to the residual factors in $\mathcal{P}_t^{\perp M}$, m1 is the inflation rate, and m2 is the unemployment gap.

μ_Z	$\Phi_Z^{(1)}$					$\Phi_M^{(2)}$		$\Phi_M^{(3)}$	
-0.070	0.994	0.055	-0.167	0.023	0.094	-0.081	0.023	-0.030	-0.006
(0.120)	(0.006)	(0.039)	(0.149)	(0.042)	(0.056)	(0.047)	(0.030)	(0.046)	(0.032)
-0.136	0.002	0.935	-0.184	-0.023	0.024	0.060	0.167	-0.051	0.009
(0.036)	(0.002)	(0.013)	(0.045)	(0.013)	(0.016)	(0.064)	(0.048)	(0.061)	(0.061)
0.033	0.002	0.011	0.862	-0.002	0.032	$\Phi_M^{(4)}$		$\Phi_M^{(5)}$	
(0.025)	(0.001)	(0.008)	(0.027)	(0.008)	(0.009)				
0.008	-0.001	-0.006	0.039	1.140	-0.046	-0.018	-0.011	-0.033	0.021
(0.014)	(0.001)	(0.005)	(0.023)	(0.036)	(0.023)	(0.041)	(0.030)	(0.043)	(0.032)
-0.046	-0.000	-0.002	0.057	-0.009	0.906	0.015	0.062	0.028	-0.045
(0.016)	(0.001)	(0.006)	(0.023)	(0.048)	(0.046)	(0.063)	(0.051)	(0.055)	(0.053)

$\Phi_M^{(6)}$		$\Phi_M^{(7)}$		$\Phi_M^{(8)}$		$\Phi_M^{(9)}$		$\Phi_M^{(10)}$	
0.049	0.005	-0.025	-0.027	0.034	0.031	0.016	0.019	-0.032	-0.008
(0.043)	(0.029)	(0.038)	(0.033)	(0.039)	(0.033)	(0.041)	(0.033)	(0.040)	(0.033)
-0.005	-0.026	-0.062	-0.068	0.042	-0.005	-0.058	0.002	0.016	-0.051
(0.055)	(0.050)	(0.058)	(0.054)	(0.058)	(0.049)	(0.062)	(0.047)	(0.057)	(0.049)

$\Phi_M^{(11)}$		$\Phi_M^{(12)}$		$\Phi_M^{(13)}$	
-0.026	-0.034	-0.239	-0.027	0.241	0.037
(0.039)	(0.031)	(0.051)	(0.030)	(0.037)	(0.023)
0.041	0.060	-0.067	-0.174	0.060	0.136
(0.053)	(0.053)	(0.061)	(0.047)	(0.042)	(0.037)

Σ_Z					$\text{diag}(A_Z^{(1)})$	$\text{diag}(A_Z^{(2)})$	$\text{diag}(B_Z^{(1)})$	$\text{diag}(B_Z^{(2)})$
0.163	-	-	-	-	0.344	0.147	0.538	0.757
(0.026)					(0.033)	(0.076)	(0.033)	(0.014)
-0.011	0.063	-	-	-	0.382	0.131	0.546	0.732
(0.011)	(0.009)				(0.059)	(0.142)	(0.042)	(0.030)
-0.017	0.014	0.031	-	-	0.338	-0.107	0.535	0.745
(0.008)	(0.007)	(0.007)			(0.053)	(0.092)	(0.074)	(0.050)
-0.027	-0.015	-0.012	0.011	-	0.196	0.182	0.482	0.798
(0.007)	(0.004)	(0.004)	(0.009)		(0.051)	(0.073)	(0.106)	(0.064)
0.007	0.032	0.021	0.061	0.000	0.088	0.326	0.636	0.514
(0.009)	(0.032)	(0.024)	(0.013)	(0.000)	(0.133)	(0.106)	(0.390)	(0.610)

Table 2: The table shows parameter estimates related to the dynamics of the state vector under the physical probability measure. Robust standard errors computed by the Huber Sandwich Estimator are reported in parentheses.

$k_\infty^{\mathbb{Q}}$	β_M'		$\mu_M^{\mathbb{Q}'}$		$\Phi_M^{\mathbb{Q}} - I_{n_M}$		$V_M^{\mathbb{Q}}$	
0.006	1.265	-0.474	-0.012	-0.602	-0.008	-0.015	1×10^{-5}	-0.001
(0.012)	(0.031)	(0.064)	(0.021)	(0.217)	(0.003)	(0.003)	(3×10^{-4})	(0.003)
					-0.011	-0.165	-0.001	0.191
					(0.028)	(0.041)	(0.003)	(0.034)

$\lambda^{\mathbb{Q}}$-1	$V_{\perp M}^{\mathbb{Q}}$		
-3×10^{-4}	1.478	0.101	-0.048
(0.002)	(0.127)	(0.102)	(0.029)
-0.037	0.101	0.198	0.031
(0.022)	(0.102)	(0.036)	(0.080)
-0.094	-0.048	0.031	0.042
(0.071)	(0.029)	(0.080)	(6.546)

Table 3: The table shows parameter estimates related to the risk-neutral probability measure. Robust standard errors computed by the Huber Sandwich Estimator are reported in parentheses.

B Extensions and Robustness Appendix

B.1 Supply and demand shock decomposition

The main paper focuses on the decomposition of variances into macroeconomic and residual shocks. This appendix decomposes the contribution of macroeconomic shocks into demand and supply shocks. To identify these shocks, I impose sign restrictions on the impulse responses as proposed by Uhlig (2005). Specifically, I identify supply shocks that decrease both the unemployment gap and inflation on impact across the entire time series, and demand shocks that decrease the unemployment gap but increase inflation. With this identification strategy, macroeconomic shocks as an entity are exactly identified, while the decomposition into supply and demand is only set-identified.

Figures 2 and 3 show the macroeconomic contributions to 1- and 12-months ahead yield curve variation decomposed into demand and supply. The figures show that the time variation in the macroeconomic shares of variation is primarily driven by demand shocks.

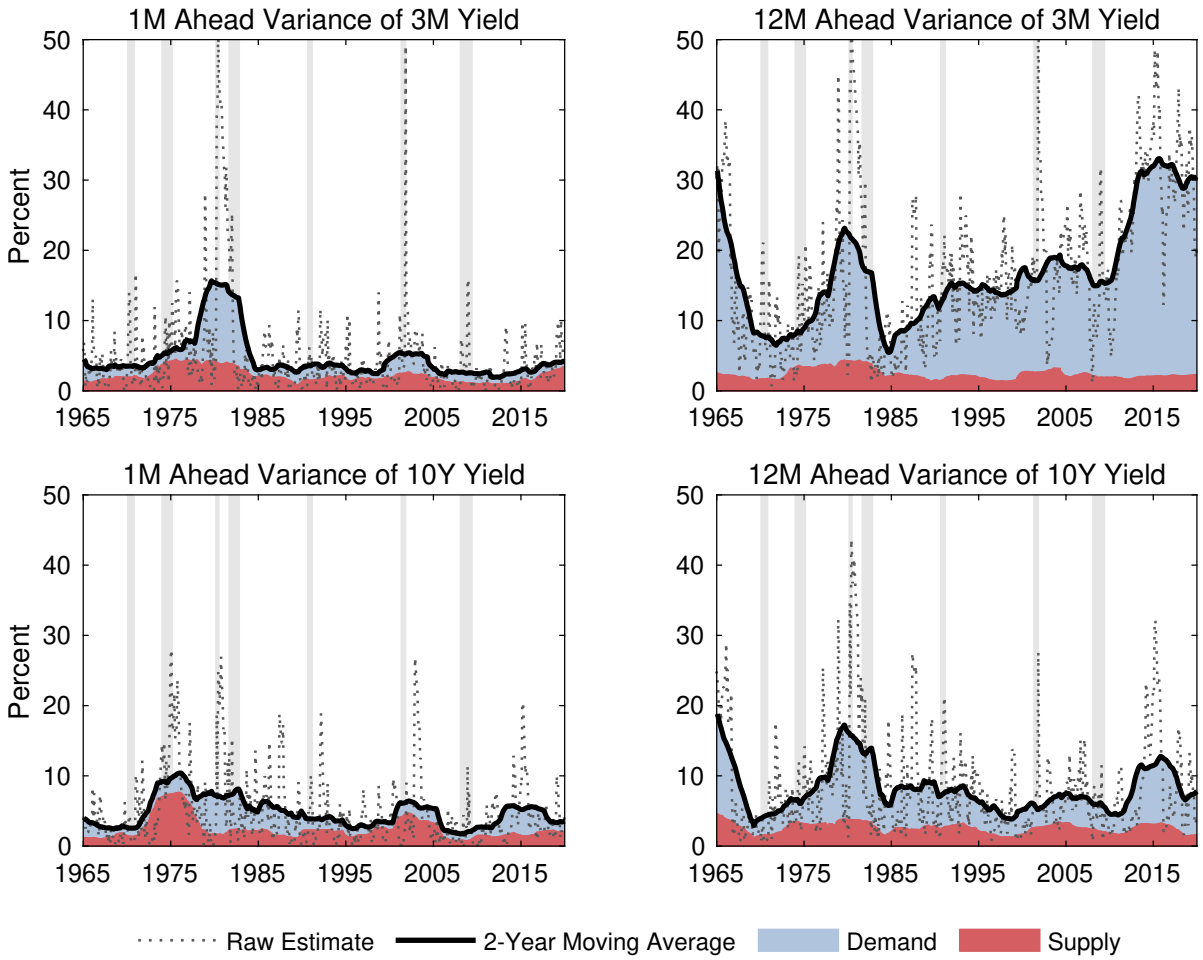

Figure 2: The figure shows the macroeconomic shares of h-step ahead conditional variances in the 3-month and 10-year yields for $h = \{1, 12\}$ months (gray dotted line). The black solid line shows the 2-year moving average, and shaded areas mark its decomposition into demand and supply shocks, computed by the median of 100 draws that satisfy the sign restrictions. Shaded areas represent NBER recessions.

B.2 Subsample analysis

Pre-crisis period: June 1961 – December 2007

I estimate the model for the period prior to the financial crisis, i.e., from June 1961 to December 2007. A model with $L = 13$ lags continues to be preferred by the data. Using the estimated parameters for the subsample, I filter the model through the financial crisis and up to December 2019. Figure 5 shows the macroeconomic shares of conditional variances in yields, short-rate expectations, and term premia. The period where the data is filtered from the model is indicated by the yellow-shaded area. For the considered subsample, the macroeconomic shares are highly similar to the decompositions obtained by the model when estimated using the full sample, up to level shifts. The macroeconomic

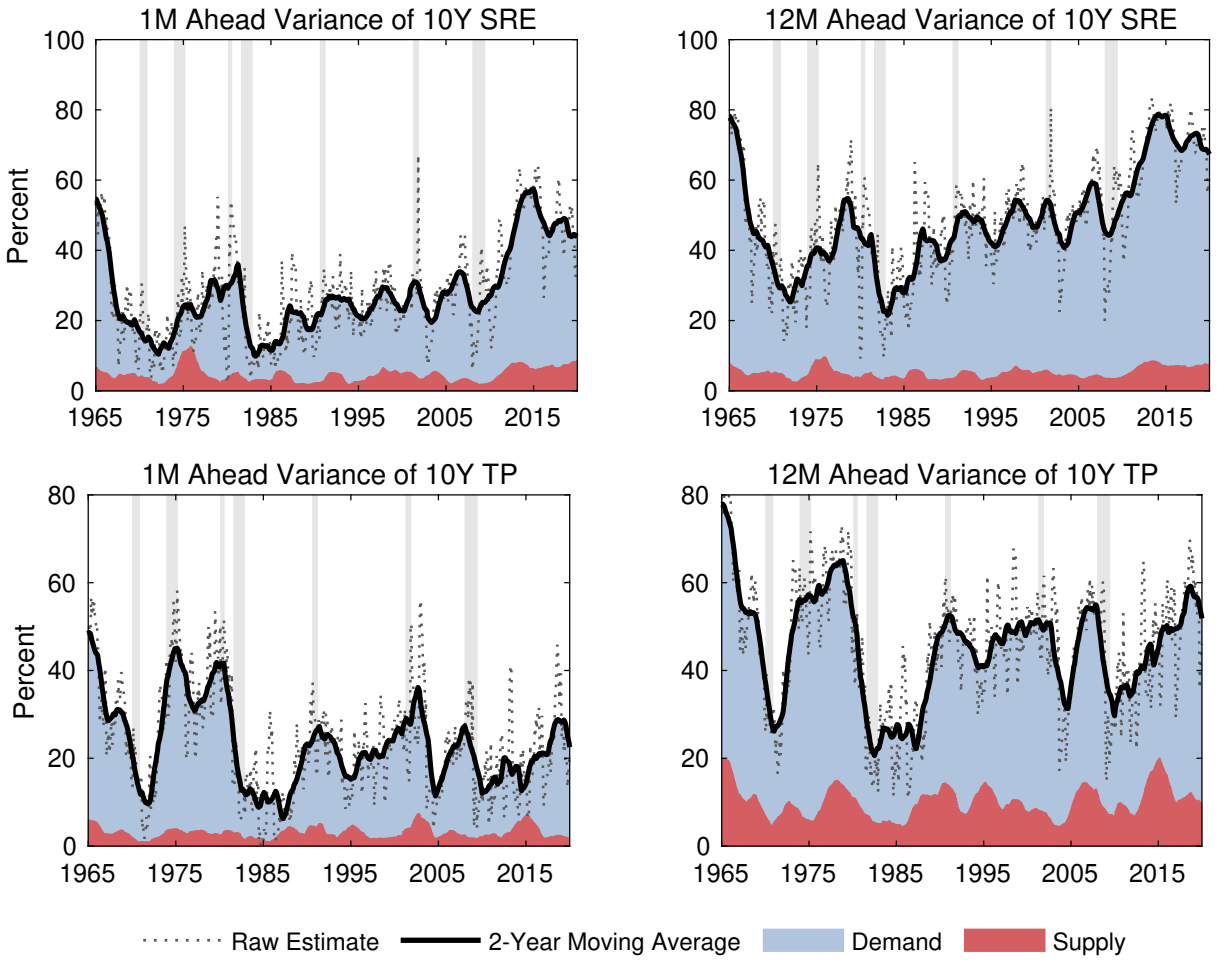

Figure 3: The figure shows the macroeconomic shares of h-step ahead conditional variances in the 10-year short-rate expectations (SRE) and term premia (TP) for $h = \{1, 12\}$ months (gray dotted line). The black solid line shows the 2-year moving average, and shaded areas mark its decomposition into demand and supply shocks, computed by the median of 100 draws that satisfy the sign restrictions. Shaded areas represent NBER recessions.

shares of one-year ahead variances computed from the subsample parameter estimates are higher for short-rate expectations and lower for term premia compared to the full-sample estimates. After the financial crisis, the filtered macroeconomic shares exhibit an increasing trend as the full-sample estimates, but the increase is more moderate.

The correlations between short-rate expectations and term premia for the subsample including the filtered results after the financial crisis are shown in Figure 4. These results are very similar to the full-sample results so that the changes in ignoring post-crisis data in estimation do not impact the estimates of this correlation.

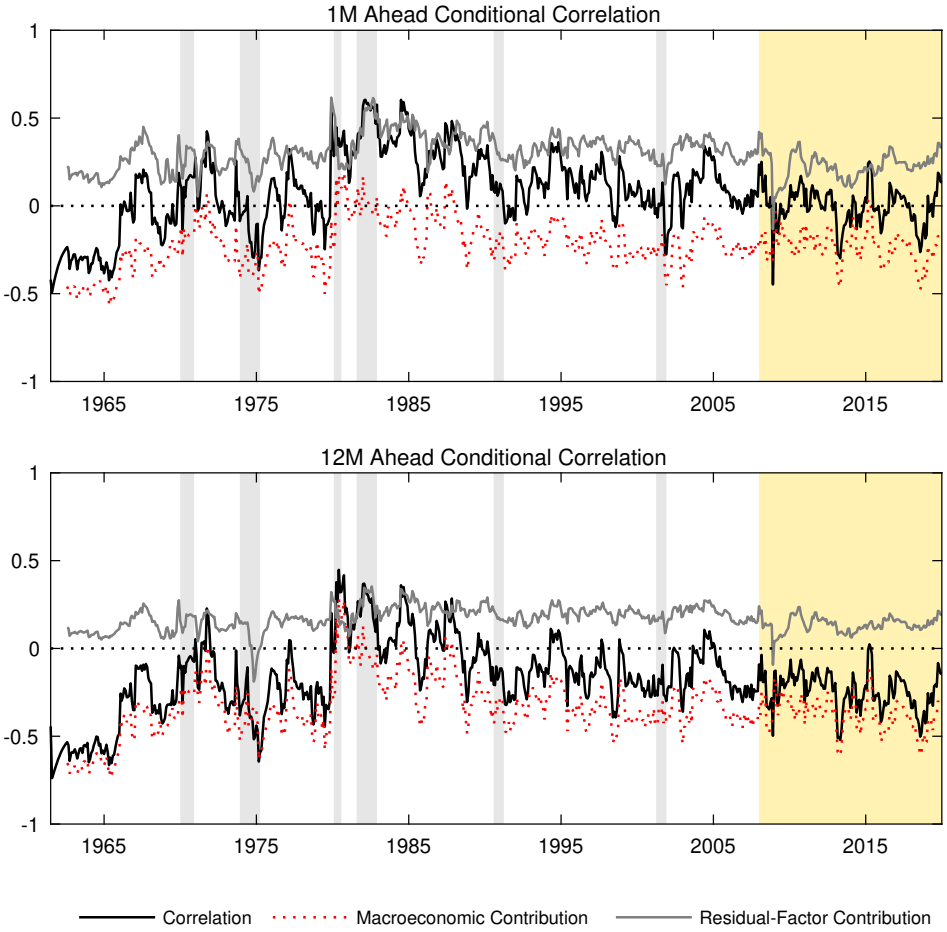

Figure 4: The figure shows the model-implied one-month and one-year ahead conditional correlation between 10-year short-rate expectations and term premia. The correlation is decomposed into a contribution from respectively macroeconomic and residual-factor shocks. Gray-shaded areas represent NBER recessions. The model is estimated using data from 1961–2007 and filtered for the remainder of the sample as marked by the yellow-shaded area.

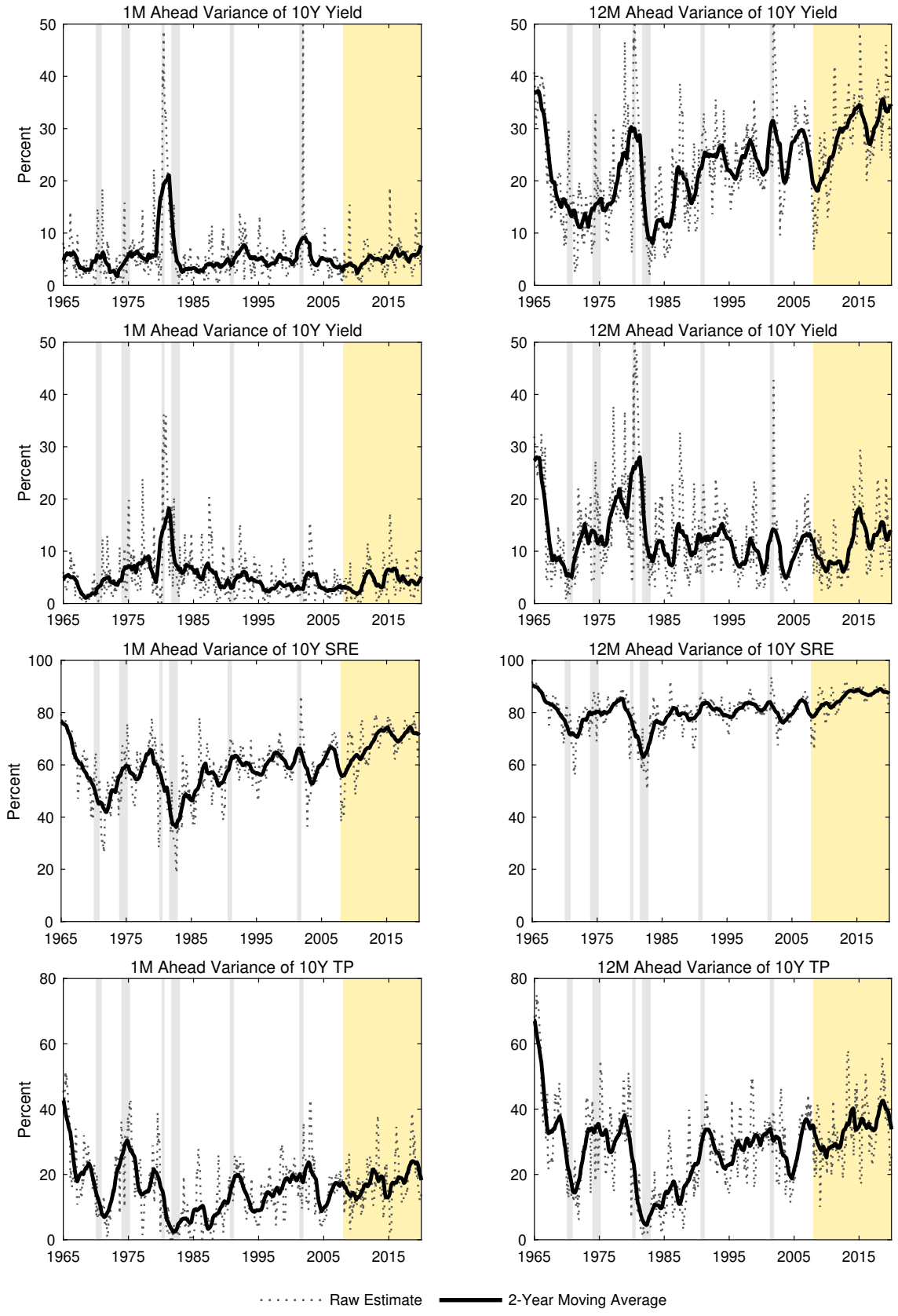

Figure 5: The figure shows the macroeconomic shares of conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their 2-year moving average (MA). Shaded areas represent NBER recessions. The model is estimated using data from 1961–2007 and filtered for the end of the sample marked by the yellow-shaded area.

The Great Moderation: August 1987 – December 2007

The subsample from August 1987 to December 2007 starts at the beginning of Alan Greenspan's chairmanship of the Federal Reserve and ends at the outbreak of the financial crisis according to the NBER. This is a short sample with 245 months and the preferred model from the full sample with $L = 13$ lags is likely to be too heavily parametrized for the data. Specification testing following the same procedure as documented for the full sample in online Appendix B.1 suggests a lag length of $L = 3$.

The variance decompositions are shown in Figure 7. I observe that the macroeconomic shares of variances are nearly constant over this sample with a few exceptions around the early 2000s recession. Table 4 shows that the average levels of the macroeconomic shares are very similar to those obtained by the Gaussian term structure model with constant conditional volatility. Therefore, for this particular subsample it is not important to account for time-varying second moments when attributing yield curve variances to macroeconomic risk.

Figure 6 shows the model-implied conditional correlations between short-rate expectations and term premia over the subsample. These are stable around 0.3 for both the one-month and one-year horizons. These results are not aligned with the correlations obtained for the considered period when including all data in the estimation. The reason may be that over the Great Moderation, the conditional variances of both yields and the macroeconomic variables are nearly constant. There is therefore no information in the data to identify potentially time-varying correlations. It continues, however, to be the

	Sample Average			Constant Conditional Volatility		
Horizons	1 month	1 year	10 years	1 month	1 year	10 years
3-month yield	5.45	21.01	25.11	4.24	26.41	21.27
1-year yield	6.23	15.89	21.69	5.14	18.90	18.16
5-year yield	2.40	9.18	21.61	1.94	10.36	19.13
10-year yield	1.72	6.83	32.05	1.46	7.52	21.66
10-year SRE	31.32	35.53	32.05	24.01	29.72	21.81
10-year TP	5.37	3.40	18.56	3.35	2.92	25.05

Table 4: The table shows average macroeconomic shares of variances in yields, short-rate expectations (SRE), and term premia (TP) compared with macroeconomic shares implied by the Gaussian term structure model with constant conditional volatility over the sample 1987–2007.

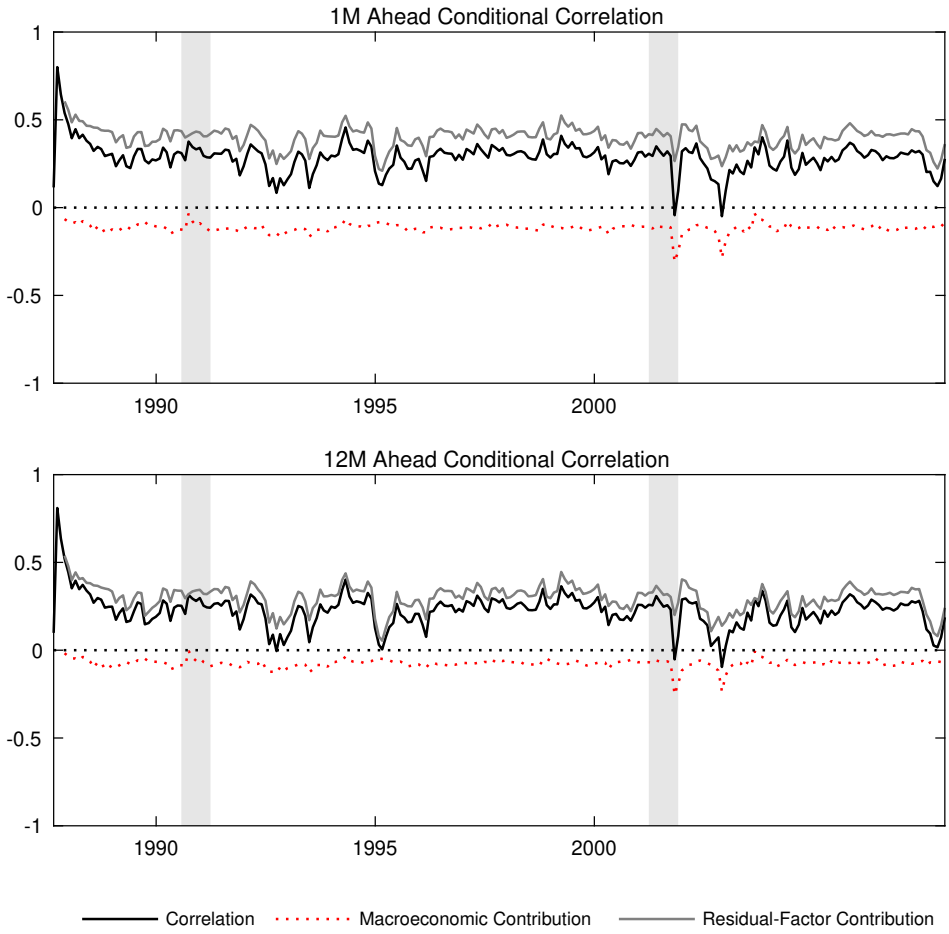

Figure 6: The figure shows the model-implied one-month and one-year ahead conditional correlation between 10-year short-rate expectations and term premia over the sample from 1987–2007. The correlation is decomposed into a contribution from respectively macroeconomic shocks and shocks to the residual factors. Gray-shaded areas represent NBER recessions.

case that the estimated positive correlation is driven by shocks to the residual factors, while macroeconomic shocks drive a negative correlation.

Figure 7: Macroeconomic Shares of Conditional Variances: 1987–2007

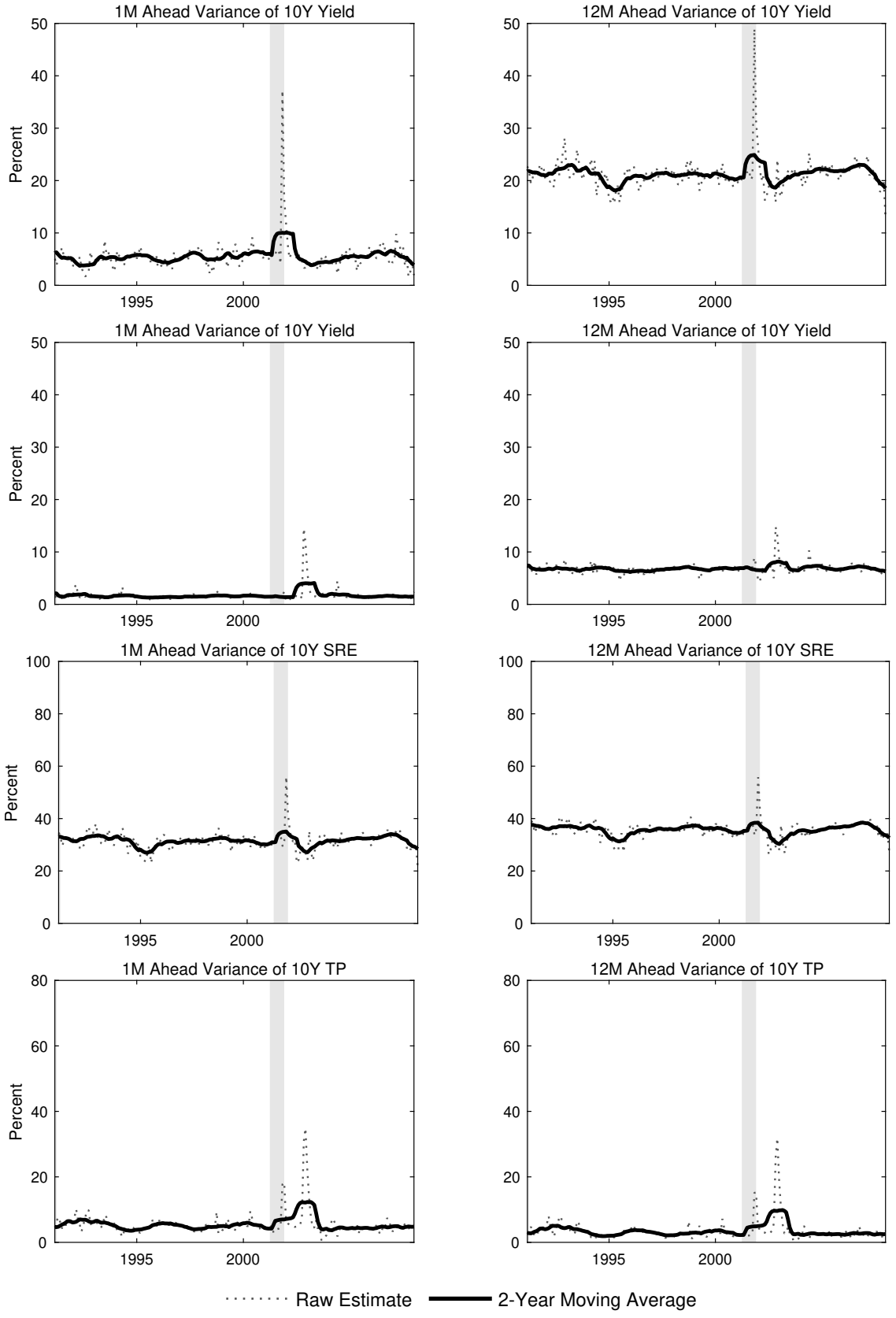

Figure 8: The figure shows the macroeconomic shares of conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their one-year moving average (MA) over the sample from 1987–2007. Shaded areas represent NBER recessions.

	Time-Varying Conditional Volatility			Constant Conditional Volatility		
Maturity	Frequency	Average (percent)	Maximum (percent)	Frequency	Average (percent)	Maximum (percent)
3-month	0.22	-0.95	-0.49	0.35	-2.23	-1.01
6-month	0.18	-0.84	-0.43	0.32	-2.08	-0.95
1-year	0.14	-0.72	-0.37	0.29	-1.93	-0.88
2-year	0.08	-0.59	-0.30	0.23	-1.76	-0.81
3-year	0.04	-0.49	-0.25	0.18	-1.62	-0.76
4-year	0.02	-0.42	-0.21	0.14	-1.49	-0.71
5-year	0.01	-0.36	-0.19	0.11	-1.39	-0.66
6-year	0.00	-0.32	-0.17	0.09	-1.30	-0.63
7-year	0.00	-0.30	-0.16	0.07	-1.22	-0.59
8-year	0.00	-0.27	-0.15	0.06	-1.15	-0.57
9-year	0.00	-0.25	-0.15	0.05	-1.09	-0.54
10-year	0.00	-0.25	-0.15	0.04	-1.03	-0.52

Table 5: The table shows results from simulating 100,000 trajectories of my model with time-varying conditional volatility and the Gaussian term structure model with constant conditional volatility during the zero-lower bound regime from December 2008 to December 2015. Frequency is the count of negative predicted yields averaged over the zero-lower bound regime and trajectories. The table also reports the average and maximum negative yields in percent predicted over the zero-lower bound regime as averages across trajectories.

B.3 The zero-lower bound regime

The zero-lower bound poses a potential problem for the model as it does not prevent negative interest rates. To understand the extent of the problem relative to the Gaussian affine term structure model, I simulate 100,000 trajectories of the models during the zero-lower bound regime from December 2008 to December 2015.

Table 5 shows the frequency of negative predicted yields and the average and maximum values of the negative yields averaged across trajectories. The table shows that when the conditional variance matrix is time varying, negative yields are predicted with a much lower frequency as compared to when conditional variances are constant. Furthermore, the simulated negative yields in my model do not reach as far into the negative territory as in the Gaussian model.

B.4 Robustness to lag length in the physical dynamics

The baseline model specification considers $L = 13$ lags in the physical dynamics. This choice is preferred by information criteria and general-to-specific likelihood-ratio tests as documented in the online Appendix B.1. Figures 9–11 show the main results for a model without an extended lag structure, i.e., for $L = 1$.

The resulting variance decompositions are shown in Figure 9. For one-month ahead conditional variances, the results are similar to the baseline model. In contrast, at the 12-month horizon, I do not observe an increasing macroeconomic contribution to variation in the 3-month yield, and the macroeconomic shares of short-rate expectations and term premia are lower compared with the predictions of the baseline model. These patterns re-emerge when considering a longer forecasting horizon, e.g., 60 months as shown in Figure 10.

The conditional correlation between short-rate expectations and term premia are shown for the model with $L = 1$ in Figure 11. It continues to hold that macroeconomic shocks drive a negative correlation, while shocks to the residual factors drive a positive correlation. However, the macroeconomic contribution is close to zero, which is a departure from the baseline results at the one-year horizon. Also, the model-implied one-year ahead correlation mainly follows the residual factor contribution for both horizons with $L = 1$. In the baseline model, the extended lag structure of the macroeconomic variables allows the macroeconomic shocks to drive the correlation at long horizons, causing the one-year ahead correlation to be negative.

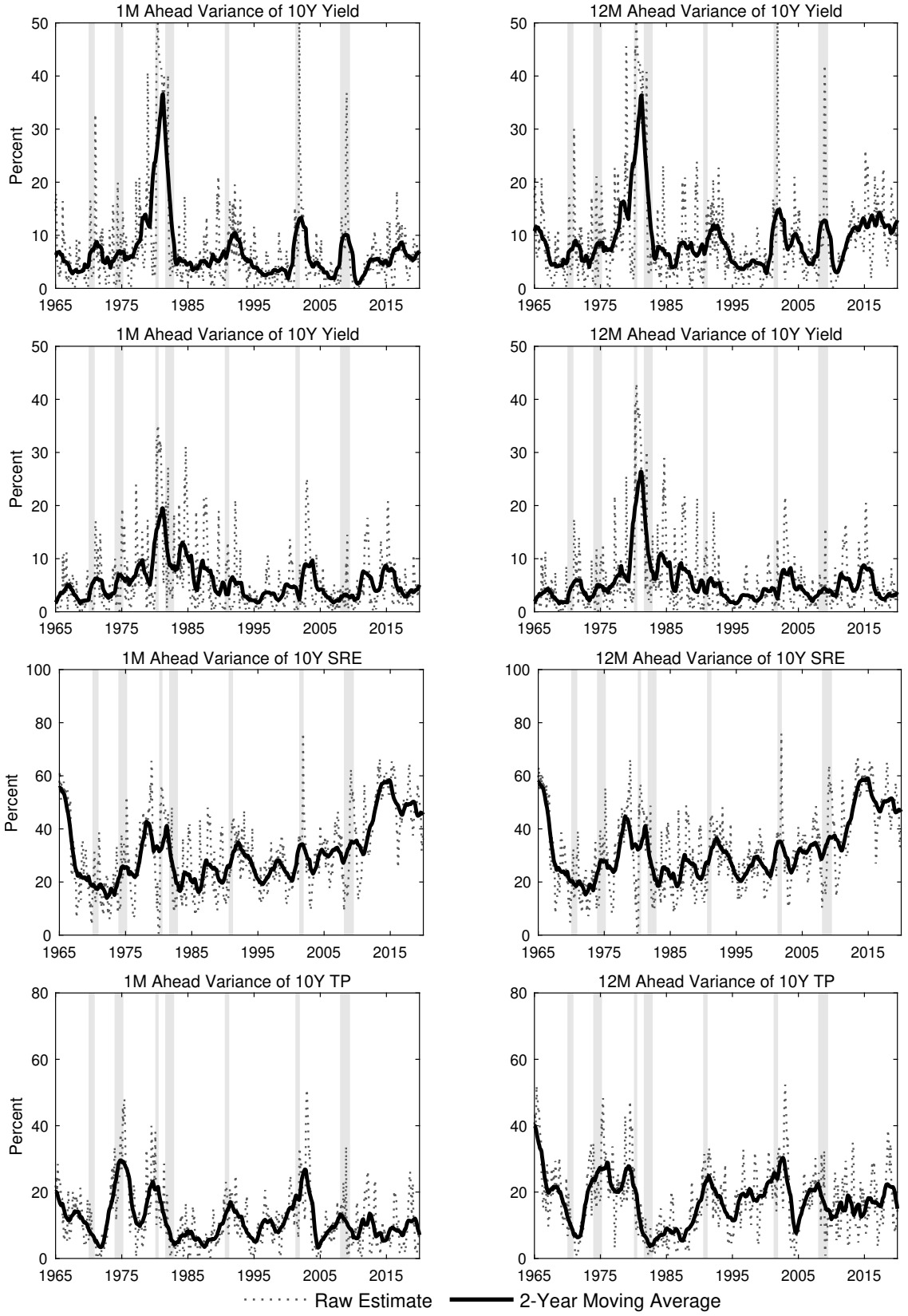

Figure 9: The figure shows the macroeconomic shares of conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their 2-year moving average (MA) generated by the model with lag length $L = 1$. Shaded areas represent NBER recessions.

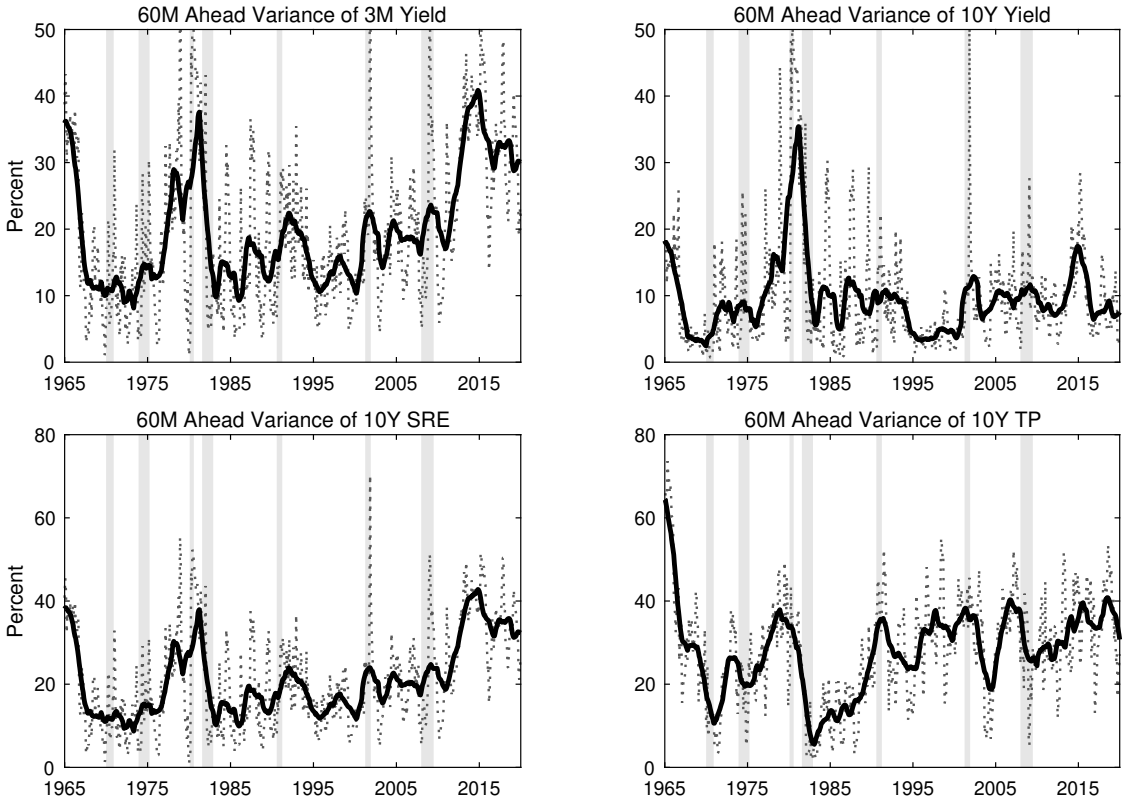

Figure 10: The figure shows the macroeconomic shares of 60-month ahead conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their 2-year moving average (MA) generated by the model with lag length $L = 1$. Shaded areas represent NBER recessions.

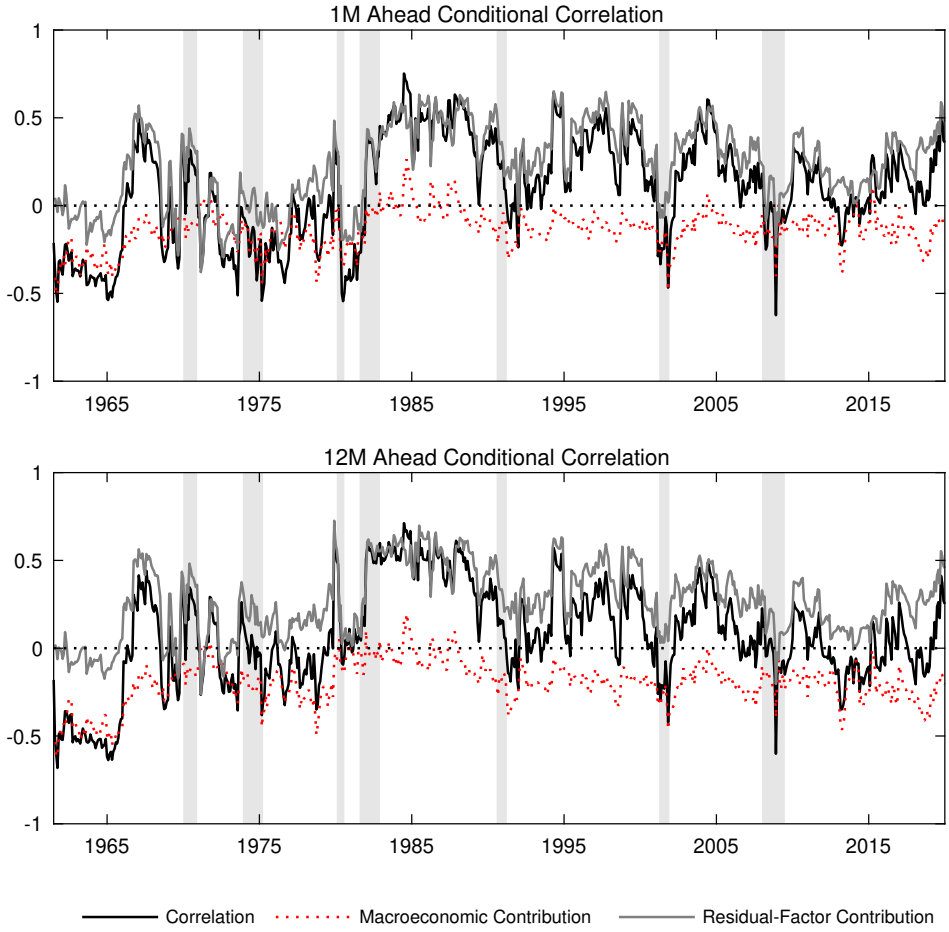

Figure 11: The figure shows the model-implied one-month and one-year ahead conditional correlation between 10-year short-rate expectations and term premia generated by the model with lag length $L = 1$. The correlation is decomposed into a contribution from respectively macroeconomic shocks and shocks to the residual factors. Gray-shaded areas represent NBER recessions.

B.5 Robustness to the choice of inflation data

In the following, I evaluate the robustness of my conclusions to the choice of inflation data. First, I consider the use of headline PCE inflation available from the Bureau of Economic Analysis. Then, I consider the year-over-year growth rate of the consumer price index (CPI), which is a common choice in the macro-finance term structure literature (Bikbov and Chernov, 2010, Cieslak and Povala, 2015). These data are available from the Bureau of Labor Statistics.

The macroeconomic shares of conditional variances are exhibited for these data choices in Figures 12–13. These results are very similar to the baseline results obtained using core PCE inflation, reported in the main paper.

Figures 14–15 show the conditional correlation between short-rate expectations and term premia with the headline inflation data. As in the baseline model, I find that macroeconomic shocks drive a negative correlation, whereas shocks to the residual factors drive a positive correlation. Unlike the baseline model with core PCE inflation, however, Figure 14 shows that with headline PCE inflation, the correlation for both the one-month and the one-year ahead horizons are generally negative. With headline CPI inflation, the one-month ahead correlation is fluctuating around zero after the Great Moderation and the one-year ahead correlation is negative. The sign of the one-month ahead correlation is thus sensitive to the choice of inflation data, whereas the one-year ahead correlation is robust.

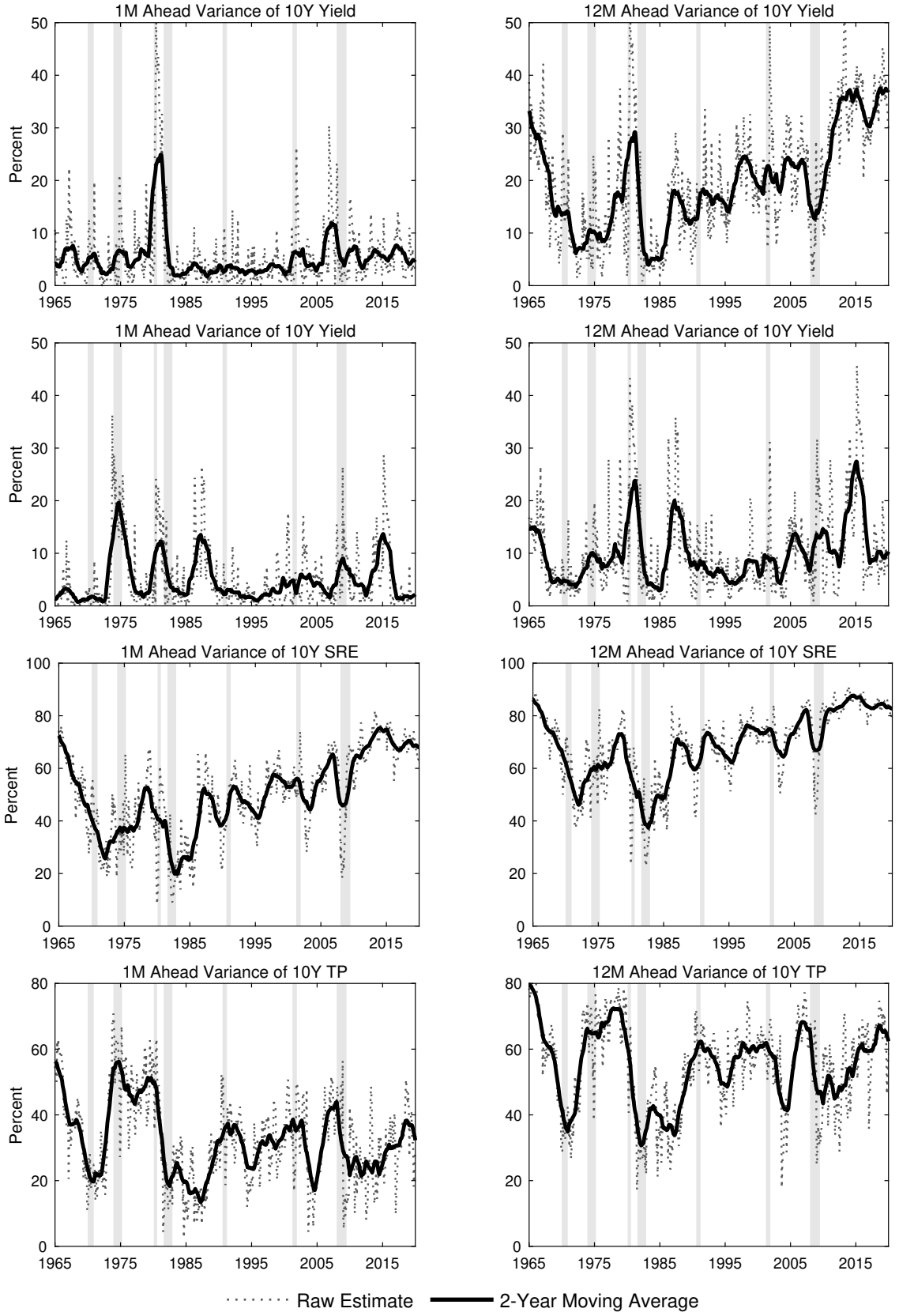

Figure 12: The figure shows the macroeconomic shares of conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their 2-year moving average (MA) generated by the model using headline PCE inflation and unemployment gap as macroeconomic variables. Shaded areas represent NBER recessions.

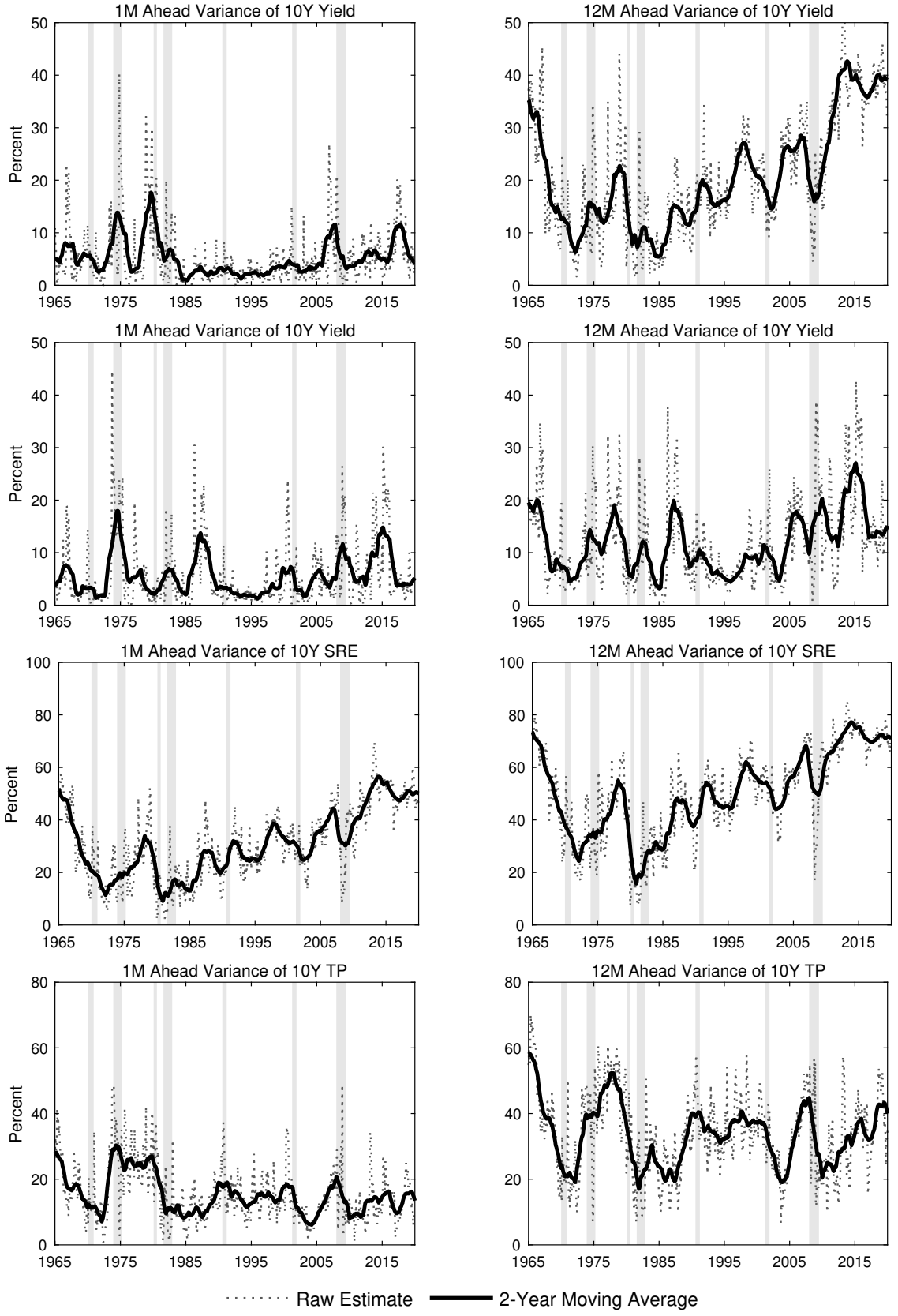

Figure 13: The figure shows the macroeconomic shares of conditional variances in yields, short-rate expectations (SRE), and term premia (TP), and their 2-year moving average (MA) generated by the model using CPI inflation and unemployment gap as macroeconomic variables. Shaded areas represent NBER recessions.

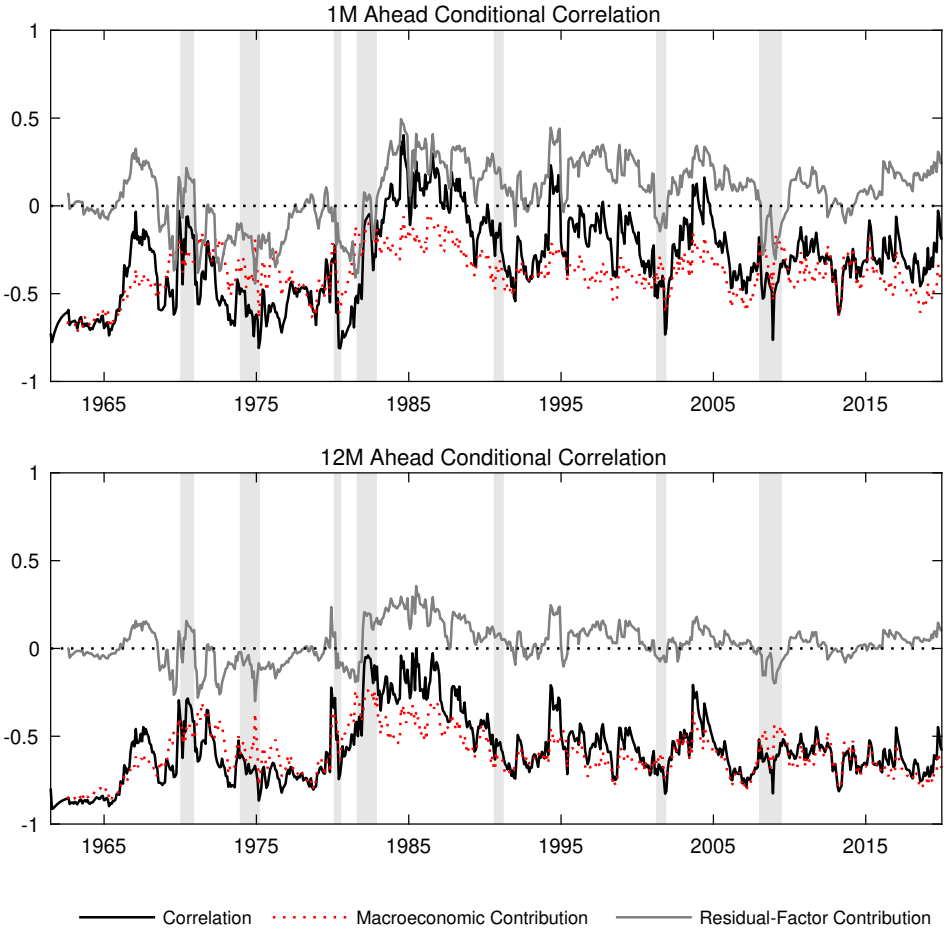

Figure 14: The figure shows the model-implied one-month and one-year ahead conditional correlation between 10-year short-rate expectations and term premia generated by the model using headline PCE inflation and unemployment gap as macroeconomic variables. The correlation is decomposed into a contribution from respectively macroeconomic shocks and shocks to the residual factors. Gray-shaded areas represent NBER recessions.

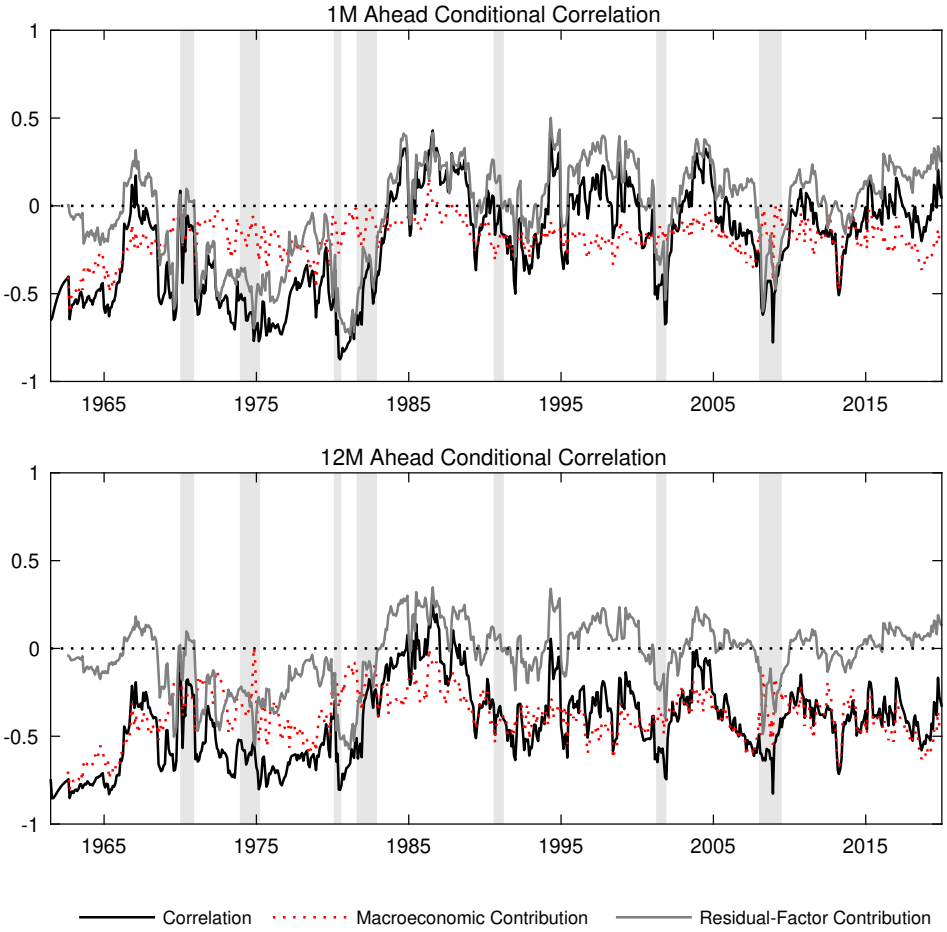

Figure 15: The figure shows the model-implied one-month and one-year ahead conditional correlation between 10-year short-rate expectations and term premia generated by the model using CPI inflation and unemployment gap as macroeconomic variables. The correlation is decomposed into a contribution from respectively macroeconomic shocks and shocks to the residual factors. Gray-shaded areas represent NBER recessions.

C Gaussian Term Structure Model

The main paper uses a model with a constant conditional variance matrix as a benchmark. This appendix presents this model, which can be described as a Gaussian Term Structure Model studied in Ang and Piazzesi (2003) among many others.

C.1 Model

The Gaussian term structure model corresponding to the setup of my model is given by the following equations:

$$r_t = \alpha_Z + \beta_M' M_t + \beta_{\perp M} \mathcal{P}_t^{\perp M}, \tag{1}$$

$$\mathcal{P}_{t+1}^{\perp M} = \mu_{\perp M}^{\mathbb{Q}} + \Phi_{\perp M}^{\mathbb{Q}} \mathcal{P}_t^{\perp M} + \varepsilon_{t+1}^{\perp m, \mathbb{Q}}, \tag{2}$$

$$M_{t+1} = \mu_M^{\mathbb{Q}} + \Phi_M^{\mathbb{Q}} M_t + \varepsilon_{t+1}^{M, \mathbb{Q}}, \tag{3}$$

$$\mathcal{P}_{t+1}^{\perp M} = \mu_{\perp M} + \Phi_{\perp M}^{(1)} \mathcal{P}_t^{\perp M} + \Phi_{\perp M, M}^{(1)} M_t + \varepsilon_{t+1}^{\perp M}, \tag{4}$$

$$M_{t+1} = \mu_M + \Phi_{M, \perp M}^{(1)} \mathcal{P}_t^{\perp M} + \Phi_M^{(1)} M_t + \Phi_M^{(2)} M_{t-1} + \ldots + \Phi_M^{(L)} M_{t+1-L} + \varepsilon_{t+1}^{M} \tag{5}$$

with $\varepsilon_t^{\mathbb{Q}} = (\varepsilon_t^{\perp M, \mathbb{Q}\prime}, \varepsilon_t^{M, \mathbb{Q}\prime})' \sim \mathcal{N}(0, V_Z)$ and $\varepsilon_t = (\varepsilon_t^{\perp M\prime}, \varepsilon_t^{M\prime})' \sim \mathcal{N}(0, V_Z)$ with V_Z block-diagonal. Defining $Z_t = (\mathcal{P}_t^{\perp M\prime}, M_t')'$, these equations can be summarized as:

$$r_t = \alpha_Z + \beta_Z' Z_t, \tag{6}$$

$$Z_{t+1} = \mu_Z^{\mathbb{Q}} + \Phi_Z^{\mathbb{Q}} Z_t + \varepsilon_{t+1}^{\mathbb{Q}}, \tag{7}$$

$$Z_{t+1} = \mu_Z + \Phi_Z^{(1)} Z_t + \Phi_Z^{(2)} Z_{t-1} + \ldots + \Phi_Z^{(L)} Z_{t+1-L} + \varepsilon_{t+1} \tag{8}$$

with appropriate restrictions on $\Phi_Z^{\mathbb{Q}}$, $\Phi_Z^{(1)}$, $\Phi_Z^{(2)}$, …, $\Phi_Z^{(L)}$. This model is distinct from my proposed model as (i) the conditional variance is constant under the physical measure and (ii) the physical and risk-neutral conditional variances are equal. Notice that the conditional variance matrix must be block-diagonal under the risk-neutral measure to ensure a proper decomposition of Y_t into the projection $\hat{Y}(M_t)$ and a residual $Y_t^{\perp M}$ orthogonal to M_t. Due to (ii), this restriction also impacts the physical conditional variance in the Gaussian model.

Equations (7)-(8) imply an exponential-linear stochastic discount factor given by

$$\mathcal{M}_{t+1} = \frac{\exp\left(-r_t + \xi_t' Z_{t+1}\right)}{\mathbb{E}_t\left[\exp\left(\xi_t' Z_{t+1}\right)\right]} \tag{9}$$

with market price of mean-based risk given by

$$\xi_t = V_Z^{-1}\left[\left(\mu_Z^{\mathbb{Q}} - \mu_Z\right) + \left(\Phi_Z^{\mathbb{Q}} - \Phi_Z^{(1)}\right) Z_t - \Phi_Z^{(2)} Z_{t-1} + \ldots + \Phi_Z^{(L)} Z_{t+1-L}\right].$$

It follows that only mean-based risk – but not variance-based risk – is priced in the Gaussian term structure model. The solution for no-arbitrage bond yields is given by (10) in the main paper with V_Z^Q replaced by V_Z. The computation of short-rate expectations and term premia is also analogous to that of the baseline model given in the main paper.

C.2 Estimation results

The Gaussian term structure model described above can be estimated by the same method as the baseline model presented in the main paper. The only difference is that the estimation of the physical dynamics can be conducted by ordinary least squares as in Joslin, Singleton, and Zhu (2011).

I estimate the model using the same setup and data as for the baseline model. The choice of $L = 13$ lags continues to be the optimal lag length evaluated as in the online Appendix B.1. Estimates of the parameters related to the physical dynamics are shown in Table 6. Parameters related to the risk-neutral dynamics coincide with those of the baseline model shown in Table 3. Therefore, the in-sample fit to yield curve levels also coincide with the baseline model.

μ_Z	$\Phi_Z^{(1)}$					$\Phi_M^{(2)}$		$\Phi_M^{(3)}$	
-0.099	0.984	-0.014	-0.040	0.027	0.048	-0.098	-0.007	-0.028	-0.083
(0.098)	(0.007)	(0.044)	(0.129)	(0.022)	(0.046)	(0.054)	(0.045)	(0.054)	(0.045)
0.130	−0.001	0.935	-0.156	-0.018	0.007	0.080	0.152	-0.031	0.014
(0.036)	(0.003)	(0.016)	(0.047)	(0.008)	(0.017)	(0.063)	(0.052)	(0.064)	(0.052)
0.045	0.001	0.014	0.840	-0.001	0.033	$\Phi_M^{(4)}$		$\Phi_M^{(5)}$	
(0.016)	(0.001)	(0.007)	(0.022)	(0.004)	(0.008)				
-0.002	-0.001	-4×10^{-4}	0.034	1.194	-0.012	-0.006	0.025	0.043	0.047
(0.012)	(0.001)	(0.005)	(0.015)	(0.034)	(0.032)	(0.054)	(0.045)	(0.054)	(0.045)
-0.034	-2×10^{-4}	-0.002	0.048	-0.023	0.958	0.033	0.015	-0.005	-0.075
(0.014)	(0.001)	(0.006)	(0.017)	(0.040)	(0.038)	(0.063)	(0.052)	(0.064)	(0.052)
$\Phi_M^{(6)}$		$\Phi_M^{(7)}$		$\Phi_M^{(8)}$		$\Phi_M^{(9)}$		$\Phi_M^{(10)}$	
0.096	0.042	-0.081	-0.020	-0.015	0.026	0.021	0.026	0.002	-0.026
(0.054)	(0.045)	(0.054)	(0.045)	(0.054)	(0.045)	(0.054)	(0.045)	(0.054)	(0.045)
-0.028	0.014	-0.076	-0.115	0.069	0.038	-0.065	0.006	0.015	-0.042
(0.064)	(0.052)	(0.064)	(0.052)	(0.063)	(0.052)	(0.063)	(0.052)	(0.064)	(0.052)
$\Phi_M^{(11)}$		$\Phi_M^{(12)}$		$\Phi_M^{(13)}$					
-0.030	-0.078	-0.457	-0.015	0.443	0.053				
(0.054)	(0.044)	(0.054)	(0.044)	(0.035)	(0.032)				
0.046	0.073	-0.029	-0.188	0.021	0.126				
(0.063)	(0.052)	(0.063)	(0.052)	(0.041)	(0.037)				

Table 6: The table shows parameter estimates related to the dynamics of the state vector under the physical probability measure in the Gaussian term structure model. Robust standard errors computed by the Huber Sandwich Estimator are reported in parentheses.

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